BEBR

FACULTY WORKING
PAPER NO. 1147

An Equal Sacrifice Solution to
Nash's Bargaining Problem

Zvi Ritz

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

BEBR

FACULTY WORKING PAPER NO. 1147

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

May, 1985

An Equal Sacrifice Solution to Nash's Bargaining Problem

Zvi Ritz, Assistant Professor
Department of Business Administration

Acknowledgement: My thanks to Abraham Neyman, Charles Blair, Bezalel Peleg, Ehud Kalai, Roger Myerson, Lloyd Shapley, and William Thomson for helpful discussions and suggestions.

AN EQUAL SACRIFICE SOLUTION TO NASH'S BARGAINING PROBLEM

by Zvi Ritz

Abstract

A new property of consistency for solutions of two-person bargaining games is introduced, the property of invariance under equivalent changes. It is demonstrated that when this property is substituted for the independence of irrelevant alternatives axiom among Nash's original axioms, it leads to a new unique solution of two-person bargaining games, a solution that allows equal amount of sacrifice by the two players.

1. Introduction

The process of reaching an agreement through the act of negotiation and bargaining is a well-established social and economic phenomenon. In a bargaining situation the two negotiating agents are faced with the problem: reach a unanimous decision--or suffer the consequences of some pre-specified disagreement outcome. For example, if the bargaining agents are the representatives of management on one hand and the employees on the other in a labor dispute, then--in case of a consensus--a new labor contract will be signed, otherwise the outcome may be a costly (to both sides) strike. Because of their enormous economic and social importance, the bargaining processes have attracted a great deal of attention.

Nash (1950) was the first to formulate the bargaining problem mathematically. In his formulation all the possible outcomes of the bargaining process, including the disagreement point, are represented as a set in the agent's utility space. This formulation enables the reduction of the analysis of bargaining situations to a search for an outcome that is agreeable to both bargainers or is considered a fair outcome by an impartial arbitrator. This outcome is referred to as the solution for the bargaining game. Nash's (1950) classical result was that there exists a unique solution that possesses the independence of equivalent utility representation, Pareto efficiency, symmetry and independence of irrelevant alternative properties defined by him. A striking aspect of Nash's solution is that it depends only on it's relation to one point--the disagreement point. Kalai and Smorodinsky (1975) replaced Nash's independence of irrelevant alternatives axiom with a monotonicity axiom and derived a unique solution that possesses the properties they considered important. Their solution depends crucially on three of the possible outcomes--the disagreement point, the outcome most preferred by the first player and the outcome most preferred by the second player. Perles and Maschler (1981) replaced the same axiom of Nash with yet another requirement, the super additivity property, and also derived a unique solution. This solution depends on the entire Pareto set of the outcome sets.

Here we consider a solution to Nash's bargaining problem that depends on all the possible outcomes. We replace the independence of irrelevant alternatives property with the invariance under equivalent changes property and demonstrate the existence of a unique solution that possesses the new property and also satisfies the rest of Nash's axioms. This solution is the outcome that allows equal amount of sacrifice to both players. This, by equating the areas of the sets of feasible outcomes which one player prefers to the solution while the other prefers the solution over them.[1]

Notations, definitions and some previous results are introduced in section 2. The new axiom and the new solution are described in section 3 and are discussed in section 4.

2. Notations and Definitions

A two-person bargaining game is represented by a pair (S,d), where S is a convex and compact subset of R^2 representing the feasible utility payoffs the two players (bargainers) can receive under cooperation, while d is an element of S corresponding to the disagreement outcome. S is assumed to contain at least one other point $x \neq d$ such that $x > d$ ($x_i > d_i$ for $i = 1,2$), thus there are incentives to both bargainers to negotiate, rather than settle immediately on the disagreement point. Denote by B the set of all such bargaining games. Let B_1 be the set of all bargaining games in B such that:

1. For every x in S, $x \geq d$, and
2. If x is in S and $d \leq y \leq x$ then y is in S.

Let B_0 be the set of all bargaining games in B_1 such that: if $d < (u_1,u_2)$ in S, then there is a pair (v_1,v_2) in S with $v_1 > u_1$ and there is a pair (w_1,w_2) in S with $w_2 > u_2$.

A solution to the bargaining problem is a function f: $B \to R^2$ such that for any (S,d) in B, f(S,d) is an element of S.

For any bargaining game (S,d), the Pareto set P(S) is defined as $P(S) = \{x \text{ in } S | \text{if } y > x \text{ then } y \text{ is not in } S\}$. Let g_S and G_S be the functions defined by

$$g_S(x) = \text{Max}\{y \text{ in } R | (x,y) \text{ is in } P(S)\}, \text{ and}$$

$$G_S(y) = \text{Max}\ \{x \text{ in } R \mid (x,y) \text{ is in } P(S)\}.$$

Since S is convex and compact, therefore both g_S and G_S are continuous monotonic decreasing functions. (Notice that for games in B_0, g_S and G_S are the inverse functions of each other.) For any bargaining game (S,d), let $I(S) = (I_1(S), I_2(S))$ be such that $I_i(S) = \text{Max}\{x_i \mid x \text{ is in } S\}$ for i = 1,2. I(S) is referred to as the <u>ideal point</u>.

We confine the discussion here to solutions which possess the following three properties introduced by Nash (1950).

<u>Independence of Equivalent Utility Representation (IEUR)</u>: For any bargaining game (S,d) and real numbers a_i and b_i for i = 1,2 such that each $a_i > 0$, let the bargaining game (S',d') be defined by $S' = \{y \text{ in } R^2 \mid \text{there exists an } x \text{ in } S \text{ such that } y_i = a_i x_i + b_i \text{ for } i = 1,2\}$ and $d_i' = a_i d_i + b_i$ for i = 1,2. Then $f_i(S',d') = a_i f_i(S,d) + b_i$ for i = 1,2.

<u>Pareto Efficiency (PE)</u>: For any bargaining game (S,d), f(S,d) is an element of P(S).

<u>Symmetry (S)</u>: For every bargaining game (S,d) such that $d_1 = d_2$ and (x_1,x_2) in S implies (x_2,x_1) is in S. Then $f_1(S,d) = f_2(S,d)$.

In addition to the above three axioms, Nash (1950), Kalai and Smorodinsky (1975) and Perles and Maschler (1981) each introduced one of the following properties as a measure of consistent behavior of solutions over related games.[2]

<u>Independence of Irrelevant Alternatives (IIA)</u> (Nash 1950): Suppose that (S,d) and (T,d) are bargaining games such that T contains S, and f(T,d) is an element of S. Then f(S,d) = f(T,d).

Individual Monotonicity (IM) (Kalai and Smorodinsky (1975)): For any two bargaining games (S,d) and (T,d) such that T contains S and $I_i(S) = I_i(T)$, either $f(S,d) = f(T,d)$ or $f_j(S,d) > f_j(T,d)$ for $j \neq i$.

Super Additivity (SA) (Perles and Maschler (1981)): For any two bargaining games (S,d) and (T,d) and for every λ, $0 \leq \lambda \leq 1$, let $\lambda S + (1-\lambda)T = \{\lambda u + (1-\lambda)b \mid u \text{ is in } S \text{ and } v \text{ is in } T\}$. Then $f(\lambda S + (1-\lambda)T,d) \geq \lambda f(S,d) + (1-\lambda)f(T,d)$.

Using the above properties the following results were derived.

Result 1 (Nash (1950))

There is a unique solution to the bargaining problem possessing the properties IEUR, PE, S and IIA. It is the function N defined by $N(S,d) = x$ such that $x \geq d$ and $(x_1 - d_1)(x_2 - d_2) > (y_1 - d_1)(y_2 - d_2)$ for all y in S such that $y \geq d$.

Result 2 (Kalai-Smorodinsky (1975))

There is a unique solution that possesses the properties of IEUR, PE, S and IM. It is the function KS defined by $KS(S,d) = u$ such that

$$(u_1 - d_1)/(I_1(S) - d_1) = (u_2 - d_2)/(I_2(S) - d_2).$$

Result 3 (Perles-Maschler (1981))

There is a unique solution for bargaining games in B_0 possessing the properties of IEUR, PE, S, SA which is also continuous. It is the function PM defined by $PM(S,d) = v$ such that

$$\int_p^v \sqrt{-du_1 du_2} = \int_v^q \sqrt{-du_1 du_2}$$

where $p = (d_1, I_2(S))$, $q = (I_1(S), d_2)$ and the integrals are the arc integrals taken along the corresponding arcs in $P(S)$.

As can be seen the Nash solution heavily depends on one feasible outcome, the disagreement point. The Kalai-Smorodinsky depends on three feasible outcomes--the disagreement outcome and outcomes $(I_1(S), d_2)$ and $(d_1, I_2(S))$, while the Perles-Maschler solution depends on the entire Pareto set.

Thomson and Myerson (1980), while investigating different properties that solutions may satisfy, defined the following twist property.

Twist: for any two bargaining games (S,d) and (T,d) and every player i, if:

(1) $f(S,d)$ is in $P(T)$

(2) x in T/S implies $x_j \leq f_j(S,d)$ for $j \neq i$

(3) x in S and $x_j \leq f(S,d)$ for $j \neq i$ implies x in T, then either

(a) $f_i(T,d) > f_i(S,d)$ or

(b) $f_i(T,d) = f_i(S,d)$ and $f_j(T,d) \leq f_j(S,d)$ for $j \neq i$.

(T/S is the set of all the outcomes which are in T but not in S.) Since all the outcomes which are in T but are not in S are outcomes which player i prefers to $f(S,d)$ while the other player prefers $f(S,d)$ over them, and all the outcomes which are in S but not in T are outcomes that player i prefers $f(S,d)$ to them while the other player prefers them over $f(S,d)$, we may say that the bargaining game (S,d) was changed in favor of player i with respect to $f(S,d)$ when the game (T,d) is considered. Thus the twist property states that a player should not suffer in the final outcome, when a bargaining situation is changed in his favor.

The twist property can be strengthened as follows. A solution satisfies the strong twist property if it satisfies the twist property and in addition if $S \neq T$ then either (a) $f_i(T,d) > f_i(S,d)$, or (b) $f_i(T,d) = f_i(S,d)$ and $f_j(T,d) < f_j(S,d)$ for $j \neq i$. Thus solutions which satisfy the strong twist are sensitive to any change in the feasible outcome set which conforms to conditions 1, 2 and 3.

None of the solutions described above satisfies the strong twist property. Consider the following example.

Example 1:

Let $S = \{x \text{ in } R^2_+ | x_1 + 2x_2 \leq 12 \text{ and } x_1 \leq 4\}$, $T = \{x \text{ in } R^2_+ | x_1 + 2x_2 \leq 12 \text{ and } 2x_1 + x_2 \leq 12\}$, and $V = \{x \text{ in } R^2_+ | x_1 + 2x_2 \leq 12 \text{ and } x_1 \leq 6\}$. Consider the three bargaining games $(S,\overline{0})$, $(T,\overline{0})$ and $(V,\overline{0})$ $(\overline{0}=(0,0))$. (See figure 1.) The Nash solutions for the games $(S,\overline{0})$ and $(T,\overline{0})$ are identical, $N(S,\overline{0}) = N(T,\overline{0}) = (4, 4)$, even though in order to obtain game $(T,\overline{0})$, only outcomes which player 1 prefers to $N(S,\overline{0})$ and player 2 prefers $N(S,\overline{0})$ to them, are added to the game $(S,\overline{0})$. Similarly, $KS(T,\overline{0}) = KS(V,\overline{0}) = (4, 4)$ when again, only outcomes which player 1 prefers to $KS(T,\overline{0})$ and player 2 prefers $KS(T,\overline{0})$ over them are added to game $(T,\overline{0})$ in order to obtain $(V,\overline{0})$. The same phenomenon, in which the solution does not positively reflect changes in a game in favor of a player, is also exhibited by the Perles-Maschler solution, since $PM(T,\overline{0}) = (4, 4)$ while $PM(V,\overline{0}) = (3, 4.5)$.

(Insert figure 1 here)

3. An Equal Sacrifice Solution

An underlying assumption for the twist property is that in order to evaluate changes in a bargaining game a player needs a reference point. The twist property is defined for the case where a game is clearly changed in favor of one of the players with respect to this reference point. The same assumption is used here for a more general case.

To motivate the approach taken here consider the following situation. Suppose two players after settling on an outcome, say x_0, as the solution for the bargaining game (S,d), are faced with (T,d) a new bargaining game with the same disagreement outcome and which also contains x_0. It stands to reason that each player while assessing his position in the new game will take into consideration the relative value, when compared to x_0, of both the set of new feasible outcomes and the set of outcomes that were feasible in the first game but are not in the second. To execute this evaluation we assume that each player uses a measure of value which he attaches to the different subsets of the feasible sets. We search for solutions which are sensitive to any change in the bargaining game as reflected in the players' valuation of the bargaining situation.

This approach can be stated formally for the most general case in the following manner. Let M_1 and M_2 be measures defined on the class of measurable sets in R^2. M_i represents the measure of value used by player i for i = 1,2. For any two bargaining games (S,d) and (T,d) and any feasible outcome x_0 in $S \cap T$, let $D_{1i}(x_0) = \{x | x \text{ in } T/S \text{ and } x_i > x_{0i}\}$, $D_{2i}(x_0) = \{x | x \text{ in } S/T \text{ and } x_i < x_{0i}\}$, $D_{3i}(x_0) = \{x | x \text{ in } T/S \text{ and } x_i < x_{0i}\}$, and $D_{4i}(x_0) = \{x | x \text{ in } S/T \text{ and } x_i > x_{0i}\}$ for i = 1,2. Notice that the addition of D_{1i} and the deletion of D_{2i} improves the position of player i with respect to x_0, while the addition of D_{3i} and the deletion of D_{4i} worsen his position. We say that player i prefers the game (T,d) to the game (S,d) with respect to outcome x_0 if $M_i(D_{1i}(x_0) \cup D_{2i}(x_0)) > M_i(D_{3i}(x_0) \cup D_{4i}(x_0))$.

Let us define the following property.

<u>General Invariance Under Equivalent Changes</u> (GIUEC) (with respect to measures M_1 and M_2): For any two bargaining games (S,d) and (T,d) in B, f(S,d) = f(T,d) = u if and only if

$$M_i(D_{1i}(u) \cup D_{2i}(u)) = M_i(D_{3i}(u) \cup D_{4i}(u)) \text{ for } i=1,2.$$

Thus, two such games may have the same solution only if each player perceives as the same the value of the set of outcomes that improve his position and the value of the set of outcomes that worsen it.

In this work we concentrate on a special case of the above property. We assume that both players use the Lebesgue measure m as their measure of value for sets of outcomes. Namely we assume that the important consideration for each player (or to an impartial arbitrator) is whether he prefers an outcome to x_0, and not by how much he prefers it.

<u>Invariance Under Equivalent Changes</u> (IUEC): For any two bargaining games (S,d) and (T,d) in B, f(S,d) = f(T,d) = u, if and only if

$$m(D_{1i}(u) \cup D_{2i}(u)) = m(D_{3i}(u) \cup D_{4i}(u)) \text{ for } i=1,2.$$

In addition, to simplify the exposition of the main idea we also restrict all the following discussions to games in B_1.

Theorem 1.

There is a unique solution to each bargaining game (S,d) in B_1 that satisfies independence of equivalent utility representation, Pareto optimality, symmetry and invariance under equivalent changes. It is the function C defined by $C(S,d) = (c_1,c_2)$ such that $c_2 = g_S(c_1)$ and

$$\int_{c_1}^{I_1(S)} [g_S(x) - d_2]dx = \int_{c_2}^{I_2(S)} [G_S(y) - d_1]dy$$

Proof.

To demonstrate that C is well defined, let (S,d) be any bargaining game in B_1 and consider the function $M(x) = A_1(x) + A_2(x)$ where

$$A_1(x) = \int_{d_1}^{x} [g_S(t) - g_S(x)]dt, \text{ and } A_2(x) = -\int_{x}^{I_1(S)} [g_S(t) - d_2]dt$$

for $d_1 \leq x \leq I_1(S)$.

Since $g_S(x)$ is a continuous monotonic decreasing function of x, therefore $A_1(x)$ is a continuous monotonic increasing function, while $A_2(x)$ is a continuous strictly monotonic increasing function. Thus $M(x)$ is a continuous strictly monotonic increasing function of x when $d_1 \leq x \leq I_1(S)$. Since $M(d_1) = A_2(d_1) < 0$ and $M(I_1(S)) \geq 0$, there exists a unique point x_0 such that $M(x_0) = 0$, or stated differently $A_1(x_0) = -A_2(x_0)$. Since $A_1(x) = \int_{d_1}^{x} [g_S(t)-g_S(x)]dt = \int_{g_S(x)}^{I_2(S)} [G_S(t)-d_1]dt$, choose $c_1 = x_0$ and $c_2 = g(x_0)$, which completes the proof that C(S,d) is well defined.

To prove that the solution C satisfies the independence of equivalent utility representation property, consider two bargaining games (S,d) and (S',d') for which there exist $a_1 > 0$, $a_2 > 0$, b_1 and b_2 such that $S' = \{y \text{ in } R^2 | \text{there exists } x \text{ in } S \text{ such that } y_i = a_i x_i + b_i \text{ for } i = 1,2\}$ and $d_i' = a_i d_i + b_i$ for $i = 1,2$. Let $C(S,d) = (c_1,c_2)$ and let $c' = (a_1c_1 + b_1,\ a_2c_2 + b_2)$.

Since $\int_{c_1'}^{I_1(S')} [g_{S'}(t) - d_2']dt = a_1a_2 \int_{c_1}^{I_1(S)} [g_S(t) - d_2]dt$, and $\int_{c_2'}^{I_2(S')} [G_{S'}(t) - d_1']dt = a_1a_2 \int_{c_2}^{I_2(S)} [G_S(t) - d_1]dt$ hence $C(S',d') = (c_1',c_2')$.

C satisfies Pareto efficiency by definition and it is simple to demonstrate that it satisfies symmetry. To prove that C possesses the invariance under equivalent changes property, observe that the conditions

$$m(D_{1i}(u) \cup D_{2i}(u)) = m(D_{3i}(u) \cup D_{4i}(u)) \quad \text{for } i=1,2$$

where m is the Lebesgue measure, can be restated as

$$\text{(i)} \quad \int_{d_1}^{u_1} g_T(x)dx - \int_{d_1}^{u_1} g_S(x)dx = \int_{u_1}^{I_1(T)} g_T(x)dx - \int_{u_1}^{I_1(S)} g_S(x)dx, \text{ and}$$

$$\text{(ii)} \quad \int_{d_2}^{u_2} G_T(y)dy - \int_{d_2}^{u_2} G_S(y)dy = \int_{u_2}^{I_2(T)} G_T(y)dy - \int_{u_2}^{I_2(S)} G_S(y)dy.$$

If in addition u is both in P(S) and P(T) then (i) and (ii) are reduced to:

$$\text{(iii)} \quad \int_{u_1}^{I_1(T)} g_T(x)dx - \int_{u_1}^{I_1(S)} g_S(x)dx = \int_{u_2}^{I_2(T)} G_T(y)dy - \int_{u_2}^{I_2(S)} G_S(y)dy.$$

It is simple to prove that C satisfies (iii). To prove the uniqueness of C, assume that H is a solution for bargaining games that satisfies all the properties stated in the theorem. Let (S,d) be an arbitrary bargaining game in B_1. Since H satisfies the IEUR property, let (S',d') be the bargaining game derived from (S,d) by changing the utility representations so that d is transformed to $d' = (0,0)$, and $h = H(S,d)$ is transformed to $h' = (1,1)$. Let $\overline{S}' = \{x \text{ in } S' | x_1 \leq x_2\}$, and $\underline{S}' = \{x \text{ in } S' | x_1 \geq x_2\}$. Since $(1,1)$ is in $P(S')$ and S' is a convex set it must be the case that either [every x in $\overline{S}'$ is such that $x_1 + x_2 \leq 2$] or [every x in $\underline{S}'$ is such that $x_1 + x_2 \leq 2$] (or both). Without loss of generality let us assume that every point x in $\underline{S}'$ is such that $x_1 + x_2 \leq 2$. Define $T = \{(x_1,x_2) \text{ in } R^2 | \text{such that } (x_2,x_1) \text{ is in } \underline{S}'\}$, and let $V = \underline{S}' \cup T$. V is a convex set, since otherwise, there exist y in $\underline{S}'$, z in T and $0 < \lambda < 1$ such that $\lambda y + (1-\lambda)z$ is not in V and $\lambda y_1 + (1-\lambda)z_1 = \lambda y_2 + (1-\lambda)z_2$. This implies $[\lambda y_1 + (1-\lambda)z_1] + [\lambda y_2 + (1-\lambda)z_2] > 2$ or $\lambda(y_1 + y_2) + (1-\lambda)(z_1 + z_2) > 2$, a contradiction. Thus (V,d') is in B_1. V is symmetric by construction therefore $H(V,d') = (1,1) = C(V,d')$. Since H satisfies invariance under equivalent changes it implies (condition (ii) above)

$$\int_1^{I_2(V)} G_V(y)dy - \int_1^{I_2(S')} G_{S'}(y)dy = \int_0^1 G_V(y)dy - \int_0^1 G_{S'}(y)dy = 0, \quad \text{and}$$

therefore $\int_1^{I_2(V)} G_V(y)dy = \int_1^{I_2(S')} G_{S'}(y)dy$. But since $C(V,d') = (1,1)$

then, $\int_1^{I_2(V)} G_V(y)dy = \int_1^{I_1(V)} g_V(x)dx$. By construction

$\int_1^{I_1(V)} g_V(x)dx = \int_1^{I_1(S')} g_{S'}(x)dx$, therefore $\int_1^{I_2(S')} G_{S'}(y)dy = \int_1^{I_1(S')} g_{S'}(x)dx$,

and hence $C(S',d') = (1,1) = H(S',d')$, which completes the proof.

Q.E.D.

Example 2.

Consider the bargaining games $(S,\overline{0})$, $(T,\overline{0})$ and $(V,\overline{0})$ defined in example 1. Then: $C(S,\overline{0}) = (10/3,13/3)$, $C(T,\overline{0}) = (4,4)$ and $C(T,\overline{0}) = (4.5,3.75)$.

4. Discussion

Solution C has the following appealing intuitive interpretation. Consider a bargaining game (S,d). Regard all the feasible points x such that $x_1 \geq c_1$ as the outcomes which the first player "gives up" in order to reach an agreement. In the same manner regard all the feasible points y such that $y_2 \geq c_2$ as the outcomes the second player "sacrifices" for the sake of reaching an agreement. Thus C is the solution that chooses the outcome that allows the same "amount of sacrifice" to both players.

The IUEC property raises a number of interesting issues. It demands that solutions to the bargaining problem be sensitive to any change in the feasible outcome set. Will this lead to attempts by the bargainers to cloud the subjects at issue by introducing essentially irrelevant outcomes to the feasible outcome set? The danger that the bargainers will try to manipulate the feasible outcome set once a solution concept was adopted exists for all the accepted solution concepts. An underlying assumption in the modeling of bargaining games (with complete information) is that both bargainers know and agree upon the feasible outcome set. Thus it is the legitimate privilege of a player to use any <u>feasible</u> outcome as an argument in the bargaining process.

Is the IUEC property too restrictive and thus point directly towards the solution C? Consider the following example of a class of solutions. For any game (S,d) in B_1 let $E_k(S,d) = (e_1,e_2)$ such that

$$e_2 = g_S(e_1) \quad \text{and}$$

$$\int_{e_1}^{I_1(S)} [g_S(t)-d_2]dt - \int_{d_1}^{e_1} [g_S(t)-g_S(e_1)]dt = k(e_1-d_1)(e_2-d_2)$$

for $-\infty < k < \infty$. Each solution E_k satisfies the IUEC property. For example $C = E_0$. Thus the set of solutions which satisfy the IUEC property is quite large. The IUEC and the GIUEC axioms do seem to imply solutions which depend on the measures used by the players. This is not necessarily a drawback, since one should expect the outcome of a bargaining process also to reflect the willingness of the players to reach a compromise and their perception of the level of cooperation of their adversary. A justification of this observation may be found in the fact that in many bargaining situations no concensus is reached because the bargaining agents perceive each other as uncooperative or inflexible.

There are a number of interesting open issues which are not discussed here. For example, the solution C possesses both the twist and the strong twist properties--is this an indication that the IUEC property is a stronger requirement than the strong twist property, namely that every solution that satisfies the IUEC also satisfies the strong twist property?

Also, in what manner should the C solution be extended to n-person bargaining games, to bargaining games in B, to games with feasible sets which are not convex? A different question is what other measures lead to solutions which satisfy the general invariance under equivalent changes property and what are their implications?

Remarks:

1. Raiffa (1953) describes an iterative solution procedure that at the end divides the feasible solution set into two equal area sets. His solution is different from the solution described here. William Thomson in a private communication indicated that he and a number of his students discussed some of the properties of the solution derived here.

2. For a detailed discussion of the axioms and definitions presented in this section, the interested reader is referred to Roth (1979) and Kalai (1983).

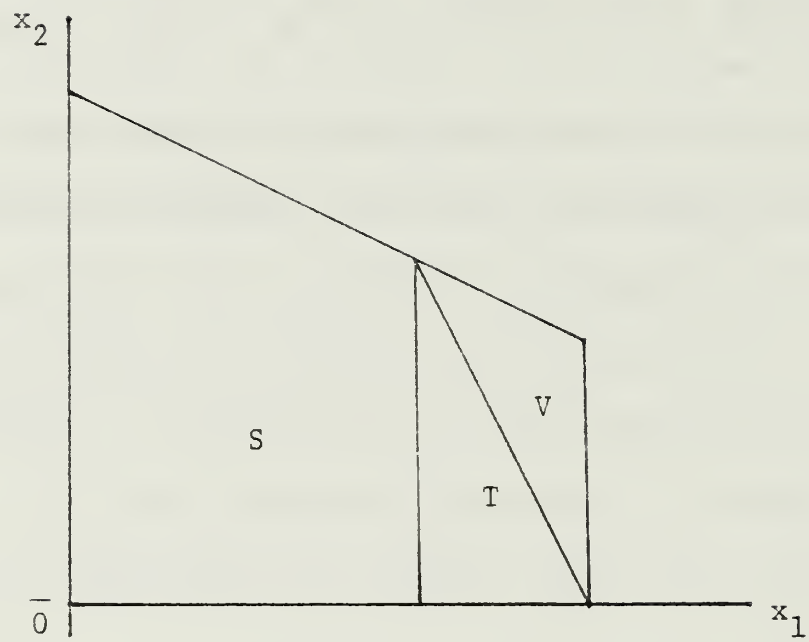

Figure 1: The bargaining games $(S,\overline{0})$, $(T,\overline{0})$ and $(V,\overline{0})$

References

Nash, J. F. (1950), "The Bargaining Problem," Econometrica, Vol. 28, pp. 155-162.

Kalai, E. and M. Smorodinsky (1975), "Other Solutions to Nash's Bargaining Problem," Econometrica, Vol. 43, pp. 513-518.

Kalai, E. (1983), "Solutions to the Bargaining Problem," Discussion Paper 556, Northwestern University.

Perles, M. and M. Maschler (1981), "The Super-Additive Solution for the Nash Bargaining Game," Int. Journal of Game Theory, Vol. 10, pp. 163-193.

Raiffa, H. (1953), "Arbitration Schemes for Generalized Two-person Games," in H. W. Kuhn and A. W. Tucker (eds.) Contribution to the Theory of Games II, Annals of Mathematics Studies, No. 28.

Roth, A. E. (1979), Axiomatic Models of Bargaining, Springer Verlag.

Thomson, W. and R. Myerson (1980), "Monotonicity and Independence Axioms," Int. Journal of Game Theory, Vol. 9, pp. 37-49.